UNITU–THEP–1/1999 FAU–TP3–99/1 hep-ph/9901322

The Infrared Behavior of Gluon, Ghost, and Quark Propagators in Landau Gauge QCD

Reinhard Alkofer, Steven Ahlig

Universität Tübingen, Institut für Theoretische Physik,
Auf der Morgenstelle 14, 72076 Tübingen, Germany
E-mail: Reinhard.Alkofer@uni-tuebingen.de
ahlig@pthp5.tphys.physik.uni-tuebingen.de

and Lorenz von Smekal

Universität Erlangen–Nürnberg,
Institut für Theoretische Physik III,
Staudtstr. 7, 91058 Erlangen, Germany
E-mail: smekal@theorie3.physik.uni-erlangen.de

February 1, 2008

PACS: 02.30.Rz 11.10.Gh 12.38.Aw 14.70.Dj

Abstract

A truncation scheme for the Dyson–Schwinger equations of QCD in Landau gauge is presented which implements the Slavnov–Taylor identities for the 3–point vertex functions. Neglecting contributions from 4–point correlations such as the 4–gluon vertex function and irreducible scattering kernels, a closed system of equations for the propagators is obtained. For the pure gauge theory without quarks this system of equations for the propagators of gluons and ghosts is solved in an approximation which allows for an analytic discussion of its solutions in the infrared: The gluon propagator is shown to vanish for small spacelike momenta whereas the ghost propagator is found to be infrared enhanced. The running coupling of the non–perturbative subtraction scheme approaches an infrared stable fixed point at a critical value of the coupling, $\alpha_c \simeq 9.5$. The gluon propagator is shown to have no Lehmann representation. The gluon and ghost propagators obtained here compare favorably with recent lattice calculations. Results for the quark propagator in the quenched approximation are presented.

1 Introduction

Despite the remarkable success of perturbative QCD the description of hadronic states and processes based on the dynamics of confined quarks and gluons remains the outstanding challenge of strong interaction physics. Especially, one has to explain why only hadrons are produced from processes involving hadronic initial states, and that the only thresholds in hadronic amplitudes are due to the productions of other hadronic states. To this end one would like to understand how singularities appear in the Green's functions of composite hadron fields where, on the other hand, they have to disappear in colored correlations functions.

To study these aspects of QCD amplitudes non–perturbative methods are required, and, since infrared divergences are anticipated, a formulation in the continuum is desirable. Both of these are provided by studies of truncated systems of Dyson–Schwinger equations (DSEs), the equations of motion of QCD Green's functions. Typically, for their truncation, additional sources of information like the Slavnov–Taylor identities, entailed by gauge invariance, are used to express vertex functions in terms of the elementary two–point functions, i.e., the quark, ghost and gluon propagators. Those propagators can then be obtained as selfconsistent solutions to non–linear integral equations representing a closed set of truncated DSEs. Some systematic control over the truncating assumptions can be obtained by successively including higher n–point functions in selfconsistent calculations, and by assessing their influence on lower n–point functions in this way. Until recently all solutions to truncated DSEs of QCD in Landau gauge, even in absence of quarks, relyed on neglecting ghost contributions completely [1, 2, 3, 4].

In addition to providing a better understanding of confinement based on studies of the behavior of QCD Green's functions in the infrared, DSEs have proven successful in developing a hadron phenomenology which interpolates smoothly between the infrared non–perturbative and the ultraviolet perturbative regime [5], for recent reviews see, *e.g.*, [6, 7]. In particular, a dynamical description of spontaneous breaking of chiral symmetry from studies of the DSE for the quark propagator is well established in a variety of models for the gluonic interactions of quarks [8]. For a sufficiently large low–energy quark–quark interaction quark masses are generated dynamically in the quark DSE in some analogy to the gap equation in superconductivity. This in turn leads naturally to the Goldstone nature of the pion and explains the smallness of its mass as compared to all other hadrons. In this framework a description of the different types of mesons is obtained from Bethe–Salpeter equations for quark–antiquark bound states [9]. Recent progress towards a solution of a fully relativistic three–body equation extends this consistent framework to baryonic bound states, see *e.g.* [10] and references therein.

Here a simultaneous solution of a truncated set of DSEs for the propagators of gluons and ghosts in Landau gauge is presented [11, 12]. An extension of this

selfconsistent framework to include quarks is subject to on–going research [13]. Preliminary results for the quark propagator in the quenched approximation have been obtained and will be shown. The behavior of the solutions in the infrared, implying the existence of a fixed point at a critical coupling $\alpha_c \approx 9.5$, is obtained analytically. The gluon propagator is shown to vanish for small spacelike momenta in the present truncation scheme. This behavior, though in contradiction with previous DSE studies [1, 2, 3, 4], can be understood from the observation that, in our present calculation, the previously neglected ghost propagator assumes an infrared enhancement similar to what was then obtained for the gluon. In the meantime such a qualitative behavior of gluon and ghost propagators is supported by investigations of the coupled gluon ghost DSEs using bare vertices [14, 15]. As expected, however, the details of the results depend on the approximations employed.

2 The set of truncated gluon and ghost DSEs

Besides all elementary 2–point functions, i.e., the quark, ghost and gluon propagators, the DSE for the gluon propagator also involves the 3– and 4–point vertex functions which obey their own DSEs. These equations involve successively higher n–point functions. The gluon equation is truncated by neglecting all terms with 4–gluon vertices. These are the momentum independent tadpole term, an irrelevant constant which vanishes perturbatively in Landau gauge, and explicit 2–loop contributions to the gluon DSE. For all details regarding this truncation scheme we refer the reader to [12].

The ghost and gluon propagators are parameterized by their respective renormalization functions G and Z,

$$D_G(k) = -\frac{G(k^2)}{k^2} \; , \quad D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \; . \tag{1}$$

In order to arrive at a closed set of equations for the functions G and Z, we use a form for the ghost–gluon vertex which is based on a construction from its Slavnov–Taylor identity (STI) which can be derived from the usual Becchi–Rouet–Stora invariance neglecting irreducible 4–ghost correlations in agreement with the present level of truncation [12]. This together with the crossing symmetry of the ghost–gluon vertex fully determines its form at the present level of truncation:

$$G_\mu(p,q) = iq_\mu \, \frac{G(k^2)}{G(q^2)} + ip_\mu \left(\frac{G(k^2)}{G(p^2)} - 1 \right). \tag{2}$$

With this result, we can construct the 3–gluon vertex according to procedures developed and used previously [16], for details see [12].

We have solved the coupled system of integral equations of the present truncation scheme numerically using an angle approximation. The infrared behavior

of the propagators can, however, be deduced analytically. To this end we make the Ansatz that for $x := k^2 \to 0$ the product $Z(x)G(x) \to cx^\kappa$ with $\kappa \neq 0$ and some constant c. The special case $\kappa = 0$ leads to a logarithmic singularity for $x \to 0$ which precludes the possibility of a selfconsistent solution. In order to obtain a positive definite function $G(x)$ for positive x from an equally positive $Z(x)$, as $x \to 0$, we obtain the further restriction $0 < \kappa < 2$. The ghost DSE then yields,

$$G(x) \to \left(g^2 \gamma_0^G \left(\frac{1}{\kappa} - \frac{1}{2} \right) \right)^{-1} c^{-1} x^{-\kappa} \quad \Rightarrow Z(x) \to \left(g^2 \gamma_0^G \left(\frac{1}{\kappa} - \frac{1}{2} \right) \right) c^2 x^{2\kappa} ,$$

where $\gamma_0^G = 9/(64\pi^2)$ is the leading order perturbative coefficient of the anomalous dimension of the ghost field. Using these relations in the gluon DSE, we find that the 3–gluon loop contributes terms $\sim x^\kappa$ to the gluon equation for $x \to 0$ while the dominant (infrared singular) contribution arises from the ghost–loop,

$$Z(x) \to g^2 \gamma_0^G \, \frac{9}{4} \left(\frac{1}{\kappa} - \frac{1}{2} \right)^2 \left(\frac{3}{2} \, \frac{1}{2-\kappa} - \frac{1}{3} + \frac{1}{4\kappa} \right)^{-1} c^2 x^{2\kappa}.$$

Requiring a unique behavior for $Z(x)$ we obtain a quadratic equation for κ with a unique solution for the exponent in $0 < \kappa < 2$:

$$\kappa = \frac{61 - \sqrt{1897}}{19} \simeq 0.92 \; . \tag{3}$$

The leading behavior of the gluon and ghost renormalization functions and thus of their propagators is entirely due to ghost contributions. The details of the approximations to the 3–gluon loop have no influence on the above considerations. Compared to the Mandelstam approximation, in which the 3–gluon loop alone determines the infrared behavior of the gluon propagator and the running coupling in Landau gauge [1, 2, 3, 4], this shows the importance of ghosts. The result presented here implies an infrared stable fixed point in the non–perturbative running coupling of our subtraction scheme, defined by

$$\alpha_S(s) = \frac{g^2}{4\pi} Z(s) G^2(s) \quad \to \quad \frac{16\pi}{9} \left(\frac{1}{\kappa} - \frac{1}{2} \right)^{-1} \approx 9.5 \; , \tag{4}$$

for $s \to 0$. This is qualitatively different from the infrared singular coupling of the Mandelstam approximation [4].

3 Comparison to lattice results

It is interesting to compare our solutions to recent lattice results available for the gluon propagator [17] and for the ghost propagator [18] using lattice versions to implement the Landau gauge condition. We would like to refer the reader

to ref. [19] where this has been done in some detail. It is very encouraging to observe that our solution fits the lattice data at low momenta rather well, especially for the ghost propagator. We therefore conclude that present lattice calculations confirm the existence of an infrared enhanced ghost propagator of the form $D_G \sim 1/(k^2)^{1+\kappa}$ with $0 < \kappa < 1$. This is an interesting result for yet another reason: In the calculation of [18] the Landau gauge condition was supplemented by an algorithm to select gauge field configurations from the fundamental modular region which is to avoid Gribov copies. Thus, our results suggest that the existence of such copies of gauge configurations might have little effect on the solutions to Landau gauge DSEs.

Here we want to add a remark concerning the comparison of the running coupling obtained in our calculation to lattice results. Recent lattice calculations of the running coupling are reported in Refs. [20, 21] based on the 3–gluon vertex, and Ref. [22] on the quark–gluon vertex. The non–perturbative definitions of these couplings are related but manifestly different from the one adopted here. One of the most recent results from the 3–gluon vertex is shown in the left graph of Fig. 1 and compared to the three–loop expression which is for the momenta displayed almost identical to our expression (4) for the running coupling. This lattice result is obtained from an asymmetric momentum subtraction scheme. This corresponds to a definition of the running coupling $\bar{g}^2_{3GVas}$ which can explicitly be related to the present one ($\bar{g}^2(t,g)$ with $t = \ln \mu'/\mu$ and $g := g(\mu)$),

$$\bar{g}^2(t,g^2)_{3GVas} = \bar{g}^2(t,g^2) \lim_{s\to 0} \frac{G^2(s)}{G^2(\mu'^2)} \left(1 - \frac{\beta(\bar{g}(t,g))}{\bar{g}(t,g)}\right)^2 . \qquad (5)$$

An inessential difference in these two definitions of the running coupling is the last factor in brackets in eq. (5) which can be easily accounted for in comparing the different schemes. However, the crucial difference is the ratio of ghost renormalization functions $G(s \to 0)/G(\mu'^2)$. These considerations show that the asymmetric scheme can be extremely dangerous if infrared divergences occur in vertex functions as our calculation indicates. Clearly, from the infrared enhanced ghost renormalization function this scale dependence could account for the infrared suppressed couplings which seem to be found in the asymmetric schemes.

Similarly, the results from the quenched calculation of the quark–gluon vertex of Ref. [22] which are compared in the right graph of Fig. 1 to our solution are obtained from an analogous asymmetric scheme. It is thus expected to have the same problems in taking the possible infrared divergences of the vertices into account which arise in both, the 3–gluon and the quark–gluon vertex, as a result of the infrared enhancement of the ghost propagator.

Furthermore, definitions of the coupling which lead to extremas at finite values of the scale correspond to double valued β–functions with artificial zeros. If the maxima in the couplings of the asymmetric schemes at finite scales are no lattice artifacts, these results seem to imply that the asymmetric schemes are

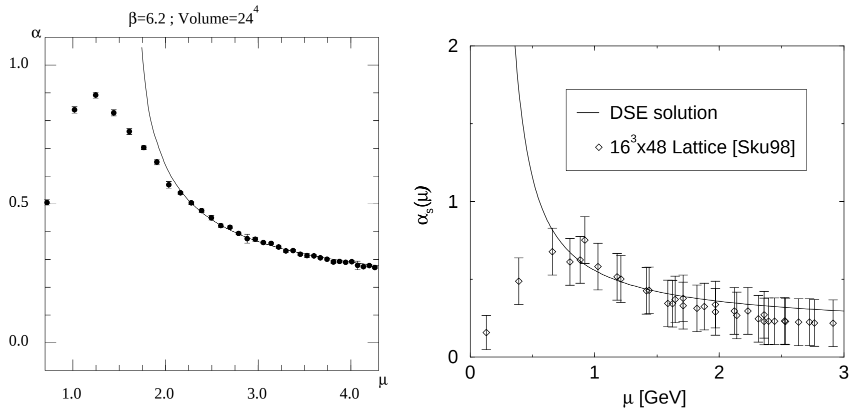

Figure 1: Lattice results of the running coupling from the 3–gluon vertex (left, together with a 3–loop fit, Fig. 1 of Ref. [21]), and from the quark–gluon vertex for $\beta = 6.0$ on a $16^3 \times 48$ lattice (right, c.f. Fig. 2 of Ref. [22]).

less suited for a non–perturbative extension of the renormalization group to all scales. Indeed, the results for the running coupling from the 3–gluon vertex obtained for the symmetric momentum subtraction scheme in Ref. [21] differ from those of the asymmetric scheme, in particular, in the infrared. These results would be better to compare to the DSE solution, however, they unfortunately seem to be much noisier thus far (see Ref. [21]).

The ultimate lattice calculation to compare to the present DSE coupling would be obtained from a pure QCD calculation of the ghost–gluon vertex in Landau gauge with a symmetric momentum subtraction scheme. This is unfortunately not available yet.

4 Quark Propagator

We have solved the quark DSE in quenched approximation [13]. In a first step we have specified the quark–gluon vertex from the corresponding Slavnov–Taylor identity. It contains explicitely a ghost renormalization function,

$$\Gamma^\mu(p,q) = G(k^2)\Gamma^\mu_{\rm CP}(p,q) \qquad (6)$$

where $\Gamma^\mu_{\rm CP}$ is the Curtis–Pennington vertex (for its definition see *e.g.* [7]). It is obvious that this leads to an effective coupling very different from the one in Abelian approximation, especially in the infrared: This effective coupling vanishes in the infrared and is similar to the lattice result of Ref. [22] shown in the right graph of Fig. 1 the main difference being that the maximum occurs at lower scale, $\mu \approx 220$MeV. This leads to a kernel in the quark DSE which is only very slightly infrared divergent. This allows, *e.g.*, to use the Landshoff–Nachtmann model for the pomeron in our approach. With our solution we obtain as Pomeron intercept 2.7/GeV as compared to the value 2/GeV deduced from phenomenology, see *e.g.* [23].

We have found dynamical chiral symmetry breaking in the quenched approximation. Using a current mass, $m(1\text{GeV}) = 6\text{MeV}$ we obtain a constituent mass of approximately 170 MeV. In the Pagels–Stokar approximation the calculated value for the pion decay constant is 50 MeV. These numbers are quite encouraging, especially for proceeding with the self–consistent inclusion of the quark DSE into the gluon–ghost system.

Considering the quark loop in the gluon DSE one realizes that the quark loop will produce an infrared divergence which is, however, subleading as compared to the one generated by the ghost loop. In the latter there appear three ghost renormalization functions in the numerator and one in the denominator leading effectively to an infrared divergence of the order $(k^2)^{-2\kappa}$. In the quark loop term there is only one factor G and thus a divergence of type $(k^2)^{-\kappa}$. Due to this subleading divergence the infrared analysis has to be redone completely before one is able to draw conclusions whether or not and how quark confinement is implemented in our set of truncated DSEs.

5 Summary

In summary, we presented a solution to a truncated set of coupled Dyson–Schwinger equations for gluons and ghosts in Landau gauge. The infrared behavior of this solution, obtained analytically, represents a strongly infrared enhanced ghost propagator and an infrared vanishing gluon propagator.

The Euclidean gluon correlation function presented here can be shown to violate reflection positivity [12], which is a necessary and sufficient condition for the existence of a Lehmann representation. We interpret this as representing confined gluons. In order to understand how these correlations can give rise to confinement of quarks, it will be necessary to redo the infrared analysis including self–consistently the quark propagator. Nevertheless, we found dynamical chiral symmetry breaking in the quenched approximation.

The existence of an infrared fixed point for the coupling is in qualitative disagreement with previous studies of the gluon DSE neglecting ghost contributions in Landau gauge [1, 2, 3, 4]. On the other hand, our results for the propagators, in particular for the ghost, compare favorably with recent lattice calculations [17, 18]. This shows that ghosts are important, in particular, at low energy scales relevant to hadronic observables.

Acknowledgments

R. A. wants to thank the organizers of the conference, and especially Dubravko Klabucar, for their help and the warm hospitality extended to him.
This work was supported by DFG (Al 279/3-1) and the BMBF (06–ER–809).

References

[1] S. Mandelstam, Phys. Rev. D **20** (1979) 3223.

[2] D. Atkinson *et al.*, J. Math. Phys. **22** (1981) 2704; D. Atkinson, P. W. Johnson and K. Stam, *ibid.* **23** (1982) 1917.

[3] N. Brown and M. R. Pennington, Phys. Rev. D **39** (1989) 2723 .

[4] A. Hauck, L. v. Smekal und R. Alkofer, preprint, ANL-PHY-8386-TH-96; Comput. Phys. Commun. **112** (1998) 149.

[5] See e.g. the contributions of D. Klabucar, C. D. Roberts and P. Tandy to these proceedings.

[6] P. C. Tandy, Prog. Part. Nucl. Phys. **39** (1997) 117.

[7] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477.

[8] V. A. Miranski, "Dynamical Symmetry Breaking in Quantum Field Theories", World Scientific, 1993, and references therein.

[9] H. J. Munczek and P. Jain, Phys. Rev. D **46** (1992) 438.

[10] M. Oettel, G. Hellstern, R. Alkofer and H. Reinhardt, Phys. Rev. C **58** (1998) 2459.

[11] L. v. Smekal, A. Hauck and R. Alkofer, Phys. Rev. Lett. **79** (1997) 3591.

[12] L. v. Smekal, A. Hauck and R. Alkofer, Ann. Phys. **267** (1998) 1.
A. Hauck, L. von Smekal, and R. Alkofer, Comput. Phys. Commun. **112** (1998) 166.

[13] S. Ahlig, L. v. Smekal and R. Alkofer, work in progress.

[14] D. Atkinson and J. Bloch, Phys. Rev. **D58** (1998) 094036.

[15] D. Atkinson and J. Bloch, Mod. Phys. Lett. A **13** (1998), 1055.

[16] S. K. Kim and M. Baker, Nucl. Phys. **B164** (1980) 152; J. S. Ball and T. W. Chiu, Phys. Rev. D**22** (1980) 2250.

[17] D. Leinweber, J. I. Skullerud and A. G. Williams, Phys. Rev. **D58** (1998) 031501.

[18] H. Suman and K. Schilling, Phys. Lett. **B373** (1996) 314.

[19] A. Hauck, R. Alkofer, and L. v. Smekal, Talk presented at the workshop "Non–perturbative Methods in Field Theory", February 2 - 13, 1998, Adelaide; to appear in the proceedings; hep-ph/9804387.

[20] B. Allés et al., Nucl. Phys. B **502** (1997) 325.

[21] Ph. Boucaud, J. P. Leroy, J. Micheli, O. Pène, and C. Roiesnel, e-prints hep-ph/9810322 and hep-ph/9810437.

[22] J. I.Skullerud, Nucl. Phys. Proc. Suppl. **63** (1998) 242.

[23] J. R. Cudell, A. Donnachie, and P. V. Landshoff, Nucl. Phys. **B322** (1989) 55.